



CLASS 12th BOARD

MATRICES



Complete Chapter in

ONE SHOT 5 PM



Ashwani
Sir

Topics To Be Covered



- ☐ Matrices and its type
- ☐ Algebra of Matrices
- ☐ Symmetric and Skew Symmetric Matrices

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Matrices

A rectangular arrangement of mn elements in the form of an ordered set of m rows, each row consisting of an ordered set of n elements is called an $m \times n$ matrix ($m \times n$ is read as m by n)

Elements $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $m \times n$

Row Coln.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{matrix} \text{--- } R_1 \\ \text{--- } R_2 \end{matrix}$$

$\downarrow \quad \downarrow$
 $C_1 \quad C_2$

Order = 2×2

$$A = \begin{bmatrix} 4 & 3 \\ 9 & 2 \\ 1 & 6 \end{bmatrix} \begin{array}{l} -R_1 \\ -R_2 \\ -R_3 \end{array}$$

$$\text{Order} = 3 \times 2$$

$m \times n$
 Row Columns

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{array}{l} -R_1 \\ -R_2 \end{array}$$

$$\text{Order} = 2 \times 3$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2 \times 3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

a_{ij}
Row Col.

$$a_{23} = 6$$

$$2 = a_{12}$$

Types of Matrices

Row Matrix:- A matrix having only one row is called a row matrix .thus $A = [1 \ 5 \ 3]_{1 \times 3}$
 $A = [1 \ 5 \ 3]$ is a 1×3 row matrix.

Column Matrix:- A matrix having only one column is called a column matrix .thus $A = \begin{bmatrix} a \\ b \\ c \end{bmatrix}_{3 \times 1}$
 $A = \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix}$ is a 3×1 row matrix.

Zero Matrix:- A $m \times n$ matrix whose each element is zero is called a zero matrix or a null matrix .thus
 $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is a zero matrix of order 2×3 .

Square Matrix:- A matrix in which the number of rows equals the number of columns say n is called a square matrix.
 $A = \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix}$ is a order 2×2 matrix. $m=n$ $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2 \times 2}$

Diagonal Matrix :- A square matrix $A = [a_{ij}]_{n \times n}$ is called a diagonal matrix .iff all its non- diagonal elements are zero.

The diagonal elements may or may not be zero

$$A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}_{2 \times 2}$$

Scalar Matrix:- A diagonal matrix in which all diagonal elements are equal is called a scalar matrix thus in a scalar matrix $A = [a_{ij}]_{n \times n}$

Example = $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

$$A = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

Identity Matrix:- A matrix in which each diagonal element is unity is called an identity matrix or a unit matrix thus in an identity matrix $A = [a_{ij}]_{n \times n}$ we have

For example $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Equal Matrices

Two Matrices are equal if

Same order

Corresponding elements Equal

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2 \times 2}$$

$$B = \begin{bmatrix} 4 & 1 \\ 6 & 2 \end{bmatrix}_{2 \times 2}$$

$$A = B$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 6 & 2 \end{bmatrix}$$

$$a = 4$$

$$b = 1$$

$$c = 6$$

$$d = 2$$

Triangular Matrix

Upper Triangular Matrix:- A square matrix $A = [a_{ij}]_{n \times n}$ is called an upper triangular matrix iff $a_{ij} = 0$ for all $i > j$ i.e. iff all element below the principal diagonal are zero.

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix}$$

Lower Triangular Matrix A square matrix $A = [a_{ij}]_{n \times n}$ is called an lower triangular matrix iff $a_{ij} = 0$ for all $i < j$ i.e. iff all element above the principal diagonal are zero.

$$A = \begin{bmatrix} 6 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 6 & 2 \end{bmatrix}$$

Note

Triangular Matrix A matrix upper or lower triangular is called a triangular matrix

Construct a 2×3 matrix whose elements a_{ij} are given by

i) $a_{ij} = 2i - j$ ii) $a_{ij} = \frac{i-j}{i+j}$ iii) $a_{ij} = \frac{(i-2j)^2}{2}$

$$a_{ij} = 2i - j$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2 \times 3} = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \end{bmatrix} \text{ Ans}$$

$$a_{ij} = 2i - j$$

$$a_{13} = 2 - 3 = -1$$

$$a_{23} = 4 - 3 = 1$$

$$a_{11} = 2 \times 1 - 1 = 1$$

$$a_{21} = 4 - 1 = 3$$

$$a_{12} = 0$$

$$a_{22} = 4 - 2 = 2$$

If $\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$ find the value of a and b

$$\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$$

$2 \times 2 \qquad \qquad \qquad 2 \times 2$

$$a+b=6 \quad \text{--- (i)}$$

$$ab=8 \quad \text{--- (ii)}$$

$$a = \frac{8}{b}$$

$$\frac{8}{b} + b = 6$$

$$8 + b^2 = 6b$$

$$b^2 - 6b + 8 = 0$$

If $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$ find the value of x, y, z and w

$$\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix}_{2 \times 2} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}_{2 \times 2}$$

$$x-y = -1$$

$$3z+w = 13$$

$$2x+z = 5$$

$$2x-y = 0$$

Operations on matrices



Addition of matrices

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 6 \end{bmatrix}_{2 \times 2} \quad B = \begin{bmatrix} 1 & 6 \\ 4 & 2 \end{bmatrix}_{2 \times 2}$$

ONLY POSSIBLE WHEN ORDER ARE SAME

$$\begin{aligned} A + B &= \begin{bmatrix} 3 & 2 \\ 1 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 4 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 3+1 & 2+6 \\ 1+4 & 6+2 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 5 & 8 \end{bmatrix} \end{aligned}$$

Subtraction of matrices

$$A = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}_{2 \times 2} \quad B = \begin{bmatrix} 6 & 8 \\ 9 & 4 \end{bmatrix}_{2 \times 2}$$

$$\begin{aligned} A - B &= \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 6 & 8 \\ 9 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 3-6 & 0-8 \\ 1-9 & 2-4 \end{bmatrix} \end{aligned}$$

$$A - B = \begin{bmatrix} -3 & -8 \\ -8 & -2 \end{bmatrix}$$

Properties of Addition.

1.) $A + B = B + A$

2.) $(A + B) + C = A + (B + C)$

3.) $A + 0 = A$

4.) $A + (-A) = 0$

Multiplication of matrix by a scalar (Number)

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

$$4A = 4 \begin{bmatrix} 4 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

$$\text{Constant } 4A = \begin{bmatrix} 16 & 12 & 8 \\ 4 & 8 & 12 \end{bmatrix} \quad \text{Ans}$$

Properties of Scalar Multiplication



$$\textcircled{k}(A+B) = Ak + Bk$$

Constant

$$(k+m)A = kA + mA$$

Matrix Multiplication

Multiplication is possible only when columns of 1st Matrix = Row of 2nd Matrix

Example-1

$$A = \left[\begin{array}{c} \\ \\ \end{array} \right]_{m \times n}$$

$$B = \left[\begin{array}{c} \\ \\ \end{array} \right]_{p \times q}$$

Example-2

$$\eta = p$$

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}$$

$\times$

$$B = \begin{bmatrix} 4 & 0 \\ 3 & 2 \\ 1 & 6 \end{bmatrix}$$

3×2

$$= \left[\begin{array}{c} \\ \\ \end{array} \right]_{3 \times 2}$$

3×3

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2 \times 3} \text{ Col.} \quad B = \begin{bmatrix} 4 & 2 \\ 0 & 1 \\ 1 & 6 \end{bmatrix}_{3 \times 2} \text{ Row}$$

$$A \times B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2 \times 3} \begin{bmatrix} 4 & 2 \\ 0 & 1 \\ 1 & 6 \end{bmatrix}_{3 \times 2}$$

$$\begin{bmatrix} 4 + 0 + 3 & 2 + 2 + 18 \\ 16 + 0 + 6 & 8 + 5 + 36 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 22 \\ 22 & 49 \end{bmatrix}_{2 \times 2}$$

Properties of Multiplication

Not Commutative $AB \neq BA$

Associative = $(AB)C = A(BC)$

Distributive = $A(B+C) = AB+AC$

$(1A=A)1=A$

Transpose of a Matrix



If $A = [a_{ij}]_{p \times q}$ be an $p \times q$ Matrix then the matrix obtained by interchanging the rows and columns of A is called the transpose of A

A^T OR A'

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & 3 \end{bmatrix} \text{---} R_1$$

$$A^T = A' = \begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 3 & 3 \end{bmatrix}$$

Properties of transpose

Double Transpose

$$[A^T]^T = A$$

Transpose of sum

$$(A+B)^T = A^T + B^T$$

Transpose of Difference

$$(A-B)^T = A^T - B^T$$

Transpose of Scalar Multiple

$$(kA)^T = kA^T$$

Transpose of Product

$$(AB)^T = B^T A^T$$

$$A = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$$

$$(A^T)^T = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$$

Symmetric Matrix:- A square Matrix if $A=(a_{ij})$ is said to be symmetric if $A^T = A$

$$A = \begin{bmatrix} 1 & 4 & 8 \\ 4 & 3 & 4 \\ 8 & 4 & 6 \end{bmatrix}_{3 \times 3}$$

$$A^T = \begin{bmatrix} 1 & 4 & 8 \\ 4 & 3 & 4 \\ 8 & 4 & 6 \end{bmatrix}$$

Skew Symmetric Matrix:- A square Matrix $A=(a_{ij})$ is said to be skew symmetric Matrix if $A^T = -A$

$$A = \begin{bmatrix} 3 & 2 & 1 \\ -2 & 4 & 9 \\ -1 & -9 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 3 & -2 & -1 \\ 2 & 4 & -9 \\ 1 & 9 & 0 \end{bmatrix}$$

$$A^T = -A$$

Theorem:- For any square Matrix A with real number entries $A+A'$ is a symmetric Matrix and $A-A'$ is a skew symmetric Matrix.

$$A = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} \quad A+A^T = \begin{bmatrix} 8 & 5 \\ 5 & 2 \end{bmatrix} \quad \text{OR} \quad A-A^T = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Theorem :- Any square Matrix can be expressed as the sum of a symmetric and a skew symmetric matrix

$$A = \frac{A+A'}{2} + \frac{A-A'}{2} \Rightarrow \frac{1}{2} [A+A' + A-A'] = \frac{1}{2} [2A] = A$$

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 6 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 4 & 3 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 6 \end{bmatrix}$$

$$A + A^T = \begin{bmatrix} 8 & 6 & 4 \\ 6 & 4 & 2 \\ 4 & 2 & 12 \end{bmatrix}$$